

OCR A Level

Computer
Science

H446 – Paper 1



Simplifying Boolean expressions

Unit 8

Boolean Algebra



PG ONLINE

Objectives

- Use the following rules to derive or simplify statements in Boolean algebra:
 - de Morgan's Laws
 - distribution
 - association
 - commutation
 - double negation
- Write a Boolean expression for a given logic gate circuit, and vice versa

Augustus de Morgan (1806 - 71)

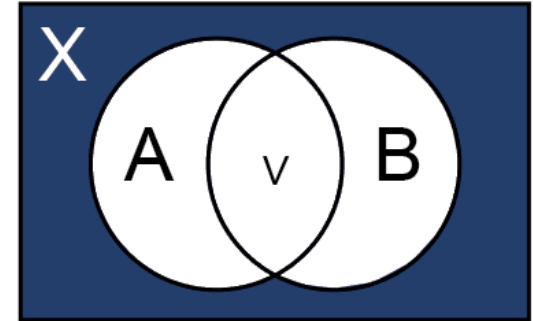
- de Morgan was a mathematician and logician who formulated laws to simplify Boolean expressions
- This had little practical use in his lifetime but became of major significance with the advent of computers



De Morgan's first law

- This states that

$$\neg(A \vee B) = \neg A \wedge \neg B$$



- Looking at the Venn diagram, the white area represents A OR B i.e. $(A \vee B)$
- X represents all the blue area, NOT (A OR B) i.e. $\neg(A \vee B)$
- The blue area is everything that is (NOT A) AND (NOT B) i.e. $\neg A \wedge \neg B$

De Morgan's first law

- Complete the truth table to show that de Morgan's Law is true, i.e. $\neg(A \vee B) = \neg A \wedge \neg B$

A	B	$A \vee B$	$\neg(A \wedge B)$
0	0		
0	1		
1	0		
1	1		

$\neg A \wedge \neg B$	$\neg B$	$\neg A$
		1
		1
		0
		0



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		1
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	1	1
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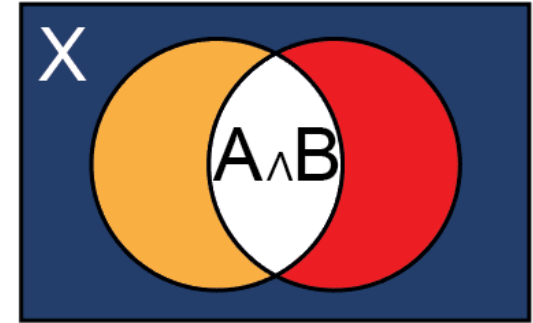
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$\neg A \wedge \neg B$	$\neg B$	$\neg A$
1	1	1
0	0	1
0	1	0
0	0	0



De Morgan's second law

- This states that



- Looking at the Venn diagram, if $X =$,
X cannot be in the white area, so it must be in
the red, orange or blue area
- That is, X is either not in A, or not in B, or not
in either
- This is the definition of $X =$

De Morgan's second law

- Complete the truth table to show that de Morgan's Law is true, i.e.

A	B	$A \wedge B$	$\neg(A \wedge B)$
0	0		
0	1		
1	0		
1	1		

$\neg A \vee \neg B$	$\neg B$	$\neg A$
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		1
		0
		0



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Implementing de Morgan's laws

1. Invert both terms in the expression e.g. $\neg P$ becomes P , $\neg Q$ becomes Q
2. Change \wedge to \vee and \vee to \wedge (AND to OR and OR to AND)
3. Invert the result

$$\text{So } \neg P \vee \neg Q = ?$$

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Simplifying expressions

- You need to be able to simplify given Boolean expressions
- As well as de Morgan's laws, there are several other "simplification rules" that will help you to do this
- See if you can fill in right hand side of the equalities on the next slide – remember, X and Y can only be either TRUE or FALSE

1 = TRUE, 0 = FALSE

Simplifying expressions

Can you figure out the following general rules?

$$1. X \wedge 0 =$$

$$2. X \wedge 1 =$$

$$3. X \wedge X =$$

$$4. X \wedge \neg X =$$

$$5. X \vee 0 =$$

$$6. X \vee 1 =$$

$$7. X \vee X =$$

$$8. X \vee \neg X =$$

$$9. \neg \neg X =$$

Simplifying expressions

Nine useful rules:

$$1. X \wedge 0 = 0$$

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Some more rules

Commutative rule

$$10. X \wedge Y = Y \wedge X$$

$$11. X \vee Y = Y \vee X$$

Associative rule

$$12. X \wedge (Y \wedge Z) = (X \wedge Y) \wedge Z$$

$$13. X \vee (Y \vee Z) = (X \vee Y) \vee Z$$

Distributive rule

$$14. X \wedge (Y \vee Z) = (X \wedge Y) \vee (X \wedge Z)$$

$$15. (X \vee Y) \wedge (W \vee Z) = (X \wedge W) \vee (X \wedge Z) \vee (Y \wedge W) \vee (Y \wedge Z)$$



Absorption rules

- There are two more rules that you will find useful for simplifying Boolean expressions:

$$X \vee (X \wedge Y) = X$$

$$X \wedge (X \vee Y) = X$$



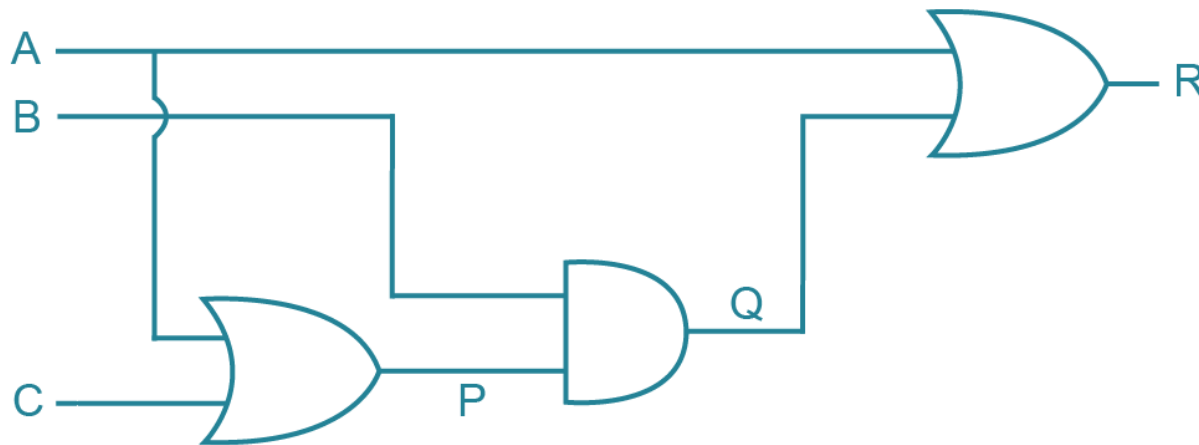
Worksheet 2

- Now try the questions in **Task 1** on **Worksheet 2**



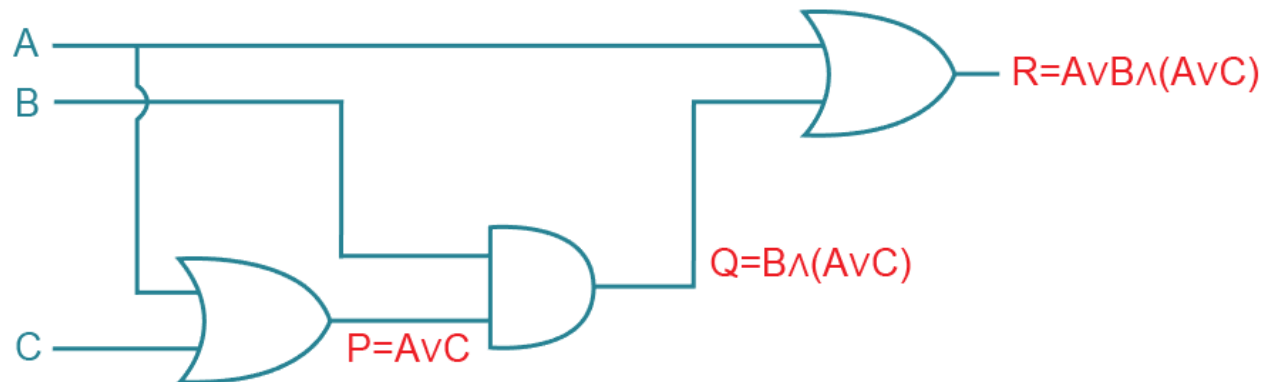
Writing expressions representing logic circuits

- Given a logic circuit, we can break it down and find the Boolean expression that it represents
- Label the output from each gate



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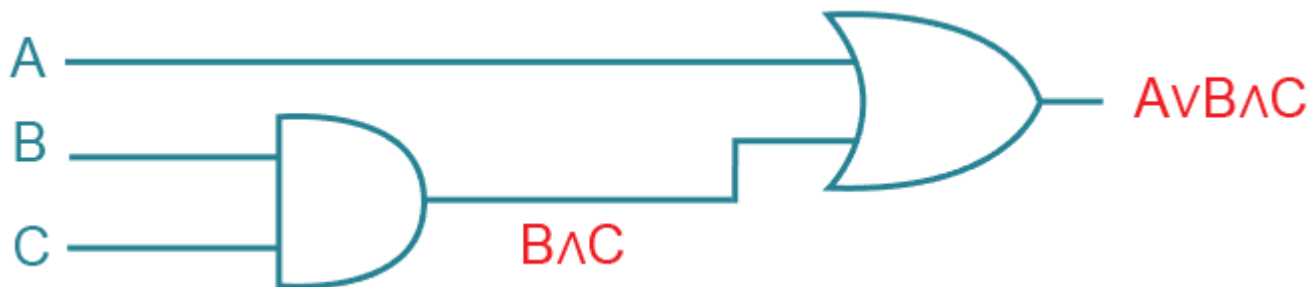
- Can you simplify R? How many gates are needed?

A simpler circuit

$$R = A \vee B (A \vee C)$$

$$= A \vee (B A) \vee (BC) \quad (\text{Distributive rule})$$

$$= A \vee (BC) \quad (\text{Absorption rule})$$



Worksheet 6

- Now try the questions in **Task 2**
- Here's an extra one to start you off:
 - Can you simplify this circuit?



Plenary

- Boolean algebra seems tricky at first
- It's all a matter of lots and lots of practice
- Try and familiarise yourself with de Morgan's Laws and the other useful rules for simplifying expressions

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